Math 18 – Final Exam

Directions: Read each problem carefully. Remember to show all work in a logical and legible manner. To receive credit, all answers must follow logically from the work that you show on your exam. You may not share calculators or use the calculator function of a cell phone.

1. Find the half-life of radioactive iodine if, after 20 days, 0.53 kilograms of an initial 3 kilograms remains. The radioactive decay formula is \( y = ae^{-bt} \).

2. Find the remaining factor(s) of \( f \) and use your results to write a complete factorization of \( f \).

\[ f(x) = 2x^3 + 3x^2 - 17x + 12, \quad \text{Factors: (2x-3), (x-1)}. \]

Remaining factor = _________________

Complete factorization of \( f \): ____________________________

3. Match the following functions with their domains. Fill in the blank with the correct letter (a-k) from the lists on the left. Not all letters will be used and some may be used more than once.

<table>
<thead>
<tr>
<th>Function</th>
<th>Domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) = 3x^3 - 3x + 3 )</td>
<td>a. ([-3, 3]) f. ([3, \infty))</td>
</tr>
<tr>
<td>( f(x) = \sqrt{x - 3} )</td>
<td>b. ((0, \infty)) g. ((-3, 3))</td>
</tr>
<tr>
<td>( f(x) = 3^{-x} )</td>
<td>c. ([0, \infty)) h. ((-3, \infty))</td>
</tr>
<tr>
<td>( f(x) = \log_3 x )</td>
<td>d. ((-\infty, \infty)) j. ((-\infty, -3))</td>
</tr>
<tr>
<td>( f(x) = \frac{3x^2}{x - 3} )</td>
<td>e. ((3, \infty)) k. none of the above</td>
</tr>
</tbody>
</table>
4. Consider the points (4, -6) and (-2, 3).
   
a. Find the slope of the line passing through these points.

   b. Find the equation of the line passing through these points. (Put your answer in slope-intercept form.)

5. A high school had an enrollment of 1200 students in 1995. During the next 10 years, the enrollment increased by approximately 50 students per year.

   a. Write a linear equation giving the enrollment N in terms of the year t. (Let t=5 correspond to the year 1995.)

   b. If this constant rate of growth continues, predict the enrollment in the year 2010.
6. Find all zeros of the following, using the methods learned in class.

   a. \( f(x) = \frac{2}{x+3} + 2(x+1) \)  
      Zeros are ________________________________

   b. \( f(x) = 2x^2 + 6x - 1 \)  
      Zeros are ________________________________

7. Consider the equation \( y = x^2 - 6x + 5 \). Find the vertex, identify it is a minimum or maximum, and find the exact (not rounded) real zeros.

   the vertex ( , ) is a ________________

   zeros: \( x = \) __________ , __________

8. Find the number of units that produces a maximum revenue \( R \) (in dollars) for \( R = 800x - 0.055x^2 \) where \( x \) is the number of units produced.
9. Let \( f(x) = \sqrt{x + 9} \), \( g(x) = x^2 + 2 \), and \( h(x) = 2x - 1 \). Find the following. Put your answers in the blanks.

a. \( f(0) = \)________________

b. \( g(x+2) = \)________________

c. \( (h - g)(x) = \)________________

d. \( (g \circ f)(x) = \)________________

e. \( (h \circ g)(-2) = \)________________

10. Solve the following system of equations using either substitution or elimination. (Choose one of the methods and show your work!)

\[
\begin{align*}
7x + 4y &= 2 \\
5x + 3y &= -4
\end{align*}
\]
11. One investment paid 8% per year and another paid 10% per year. If $4,000 was split between the two and earned exactly $368.73 total interest in a year, how much was invested at each rate?

Amount invested at 8% = ______________

Amount invested at 10% = ______________

12. Write the expression as a single logarithm and simplify as much as possible.

\[ \log_7(2x) - [\log_7(x-1) + 2\log_7(x)] \]

13. Expand the logarithmic expression as much as possible.

\[ \log_3 \left( \frac{5x^3}{\sqrt{x-1}} \right) \]
14. Solve for x. Round to four decimal places if necessary. Put your answers in the blanks.

a. $11 = 3^{(x+7)}$  
   $x=$_______________________

b. $\ln \sqrt{x+1} = 2$  
   $x=$_______________________

c. $\frac{600}{1+e^{-x}} = 575$  
   $x=$_______________________

d. $\log_3 x + \log_3 (x-8) = 2$  
   $x=$_______________________
15. If after \( w \) weeks of training, Maria rides a horse at \( S = 60(1 - e^{-0.08w}) \) km per hour, how many weeks will it take her to ride at 20 km per hour?

16. On the day a child was born, a lump sum was deposited in a trust fund paying 6.5\% interest compounded continuously. How much money should have been deposited so that the trust will be worth 1,000,000 on the child's 25\textsuperscript{th} birthday?

Bonus: A group of 642 CIA agents is captured by a terrorist cell. They plan on killing every other agent (every second agent) until only one is left. Which is the best seat to sit in?