Hypothesis Testing Worksheet #2

I. State Ho/HA (null and alternative hypothesis)
   Ho: This year's model has not changed its avg. fuel efficiency (μ) & so it is still 21 mpg (μ = 21).
   Ha: This year's model has improved fuel efficiency so that now μ > 21.

II. Possible Mistakes.
   Type I: State HA when actually Ho is correct. This is saying we have a more fuel efficient car when we
   really don't. **(False advertising)**
   Type II: This is sticking with the null hypothesis when we should have rejected it. This is saying that
   the new car is not more fuel efficient when in fact it is. **Missed opportunity** to advertise we did better

Evaluating these errors is to some degree a matter of opinion, but Type I may generally be seen
   to be worse. This is a case where the significance level should probably be set at 1% or 5%.

III. Significance Level
   Based on the discussion above we will protect against false advertising and set α as threshold
   at 1%

IV. Test Statistic / Table
   n = 44 here, so the CLT (Central Limit Theorem) applies & tells us X is approx. normal. We
Should use Table N in these calculations. We will have to replace unknown \( \sigma \) with \( s = 8.8 \), however.

Our test statistic formula will be:

\[
Z = \frac{\bar{X} - \mu}{s / \sqrt{n}}
\]

V. Plug in Step.

Our calculated test statistic is:

\[
Z = \frac{23.3 - 21}{8.8 / \sqrt{44}} = 1.73
\]

VI. P-value Step.

We got \( Z = 1.73 \).

Table N tells us that the chance of getting this value or one more extreme (which is p-value) if null is true is:

\[
P(Z > 1.73) = 1 - .9582 = .0418
\]

VII. Our Conclusion

The p-value (.0418) is not less than our significance level threshold (.01) & so we cannot reject the null hypothesis. We must stick with the null & conclude that this year's model is not better than last. (If 190 chosen in III)

Notice that if we had chosen a threshold to be 590, we would have (barely) sided with the alternate. This is an example of decisions being driven by fear of mistakes!
Hypothesis Testing Worksheet #3

I. State Null and Alternative Hypothesis.

$H_0$ (Null): Soil is still in compliance so that the avg. parts per billion of the chemical is 5 or less ($\mu \leq 5$).

Alternative: Soil is contaminated & has changed to have elevated chemical levels ($\mu > 5$).

II. Possible Mistakes

Type I error: Stating the alternative when in fact the null hypothesis is correct. This is saying the soil is contaminated when actually it is still in compliance.

Type II error: Sticking with the null hypothesis when in fact the alternative is correct. This is saying the soil is in compliance when in fact it is contaminated.

III. Significance Level.

The Type II error will be thought of as more hazardous to citizens' health & well-being. We will have created a false sense of security which could prove very dangerous to people. Claim Type II is must severe & so choose a significance level (threshold) to be 10%.

IV. Test Statistic/Table.

We have only seven observations and we must calculate $s$ ourselves, so surely $\sigma$ is unknown. Assuming our sample comes from a normally distributed population allows us to use Table I.
Our test statistic for this problem will be
\[ t = \frac{\overline{X} - \mu}{s \sqrt{n}}. \]

V. Plug-In Step.
Here, we have to calculate \( \overline{X} \) and \( s \) ourselves from the data given.

\[ \overline{X} = 36.3/7 = 5.186. \]

Using the shortcut formula for \( s^2 \):

\[ s^2 = \frac{\sum x^2 - (\overline{X})^2}{n-1} = \frac{189.4084 - (36.3)^2/7}{6} \]

\[ = 0.1945, \] so \( s = \sqrt{0.1945} = 0.441. \)

Now, \( t = \frac{5.186 - 5}{0.441/\sqrt{7}} = 1.116 \)

VI. P-value
Looking at Table T with \( df = 8 \), 1.116 is very close to the "1.1081" in the table. This tells us that the p-value is \( \approx 0.150 \) (1.1081 is in the .150 column).

VII. Conclusion
The p-value (.15) is not less than our significance level threshold (.10) so we cannot reject the null hypothesis. We must stick with the null and conclude the soil is still in compliance.

(Surprised? This is a case where we have some evidence for the alternate, but not conclusive evidence for it.)