MATH 144 Summer I 2009
PRACTICE FINAL EXAM

1. $\lim_{x\to 1} \dfrac{x^2+2x-3}{x-1} =$

2. Is $f(x) = \begin{cases} x^2-7 & if \quad x \le 4 \\ 2x+1 & if \quad x > 4 \end{cases}$ continuous? Why or why not?

3. Write the equation of the tangent line to $y = \dfrac{x^2-7}{x+1}$ at $x = 1$.

4. Find the derivatives of the following functions:
   (a) $f(t) = (t^4 + 3t)^{18}$

   (b) $y = \ln(7x^3 - 2x^2 + 3)$

(c) $g(x) = x^4 e^{5x}$

(d) $y = \dfrac{7e^{2x} + 4}{x^3 + 4x + 5}$

5. Find the **third derivative** of $y = 4x^5 - 6x^3 + 3x$.

6. If $x^4 y + 2x^2 - 3y^3 = 1$, find $\frac{dy}{dx}$.

7. Sketch the graph the function $y = x^4 - 4x^3 + 2$, labeling all critical points and inflection points. Justify your work.

8. Find the horizontal and vertical asymptotes of the following functions:

(a) $f(x) = \dfrac{2x+2}{(x+1)(x-2)}$

(b) $g(x) = \dfrac{3x^2}{x^2+x-2}$

9. Sketch the graph of the function $f(x) = \dfrac{x^2}{x^2 - 1}$, identifying any asymptotes. Label the axes of your graph.

10. Evaluate the following integrals:

(a) $\int (x^2 + 6x)^8 (x + 3)\ dx$

(b) $\int \frac{x^3 - 2}{x^4 - 8x}\ dx$

(c) $\int 11xe^{4x^2}\ dx$

11. Suppose a rancher wants to enclose a pen having total area $150m^2$ and a divider across the middle. If fencing costs \$10 per meter, what dimensions will minimize the cost of building the pen?

12. Find the general solution to $x^5y^3\ dx = dy$.

13. $\sum_{k=1}^{85}(k^2 - 5k + 6) =$

14. Consider the function $f(x) = x^2 - 3x + 4$.

 (a) What is the **approximation** of the area under the curve from $\mathbf{x = 0}$ to $\mathbf{x = 2}$ using **right** endpoints and **4 subintervals**?

 (b) What is the **exact** area?

15. Evaluate the following definite integrals:

 (a) $\int_0^2 3x^2 - 6x + 5 \quad dx$

(b) $\int_0^1 x^2(x^3+1)^4 \quad dx$

16. Find the area enclosed by $y = 5x + 1$ and $y = x^2 + 2x + 3$.

17. What is the average value of $f(x) = 6x^2 - 2x$ from 0 to 5?

18. Suppose that the monthly revenue and cost (in dollars) for x units of a product are given by $R = 400x - \frac{x^2}{20}$ and $C = 5000 + 7x$. At what rate per month is the profit changing when the number of units produced

and sold is 100 and is increasing at a rate of 10 units per month?

19. Suppose that marginal revenue for a product is given by $\overline{MR} = 10x + 20$, its marginal cost is $\overline{MC} = x^2 + 2x$, and the cost to produce 3 items is \$120. Find the following:

   (a) Optimal level of production

   (b) Revenue function

   (c) Total cost function

(d) Fixed costs

(e) Profit function

(f) Profit at the optimal level of production

20. If the marginal cost for a product is $\overline{MC} = 90\sqrt{2x+1}$ and fixed costs are \$100, find the total cost function.

21. Integrate the following using the **Table of Integrals** provided:

(a) $\int \frac{3dx}{x\sqrt{4-x^2}}$

(b) $\int \frac{dx}{25-(3x+1)^2}$

22. Integrate the following:

(a) $\int 10x^3(x^2+7)^4 \, dx$

(b) $\int_0^{\infty} e^{-3x} dx$

(c) $\int_1^{\infty} x^5 dx$

Table of Integrals

1.

$$\int \frac{du}{u(au+b)} = \frac{1}{b} \ln \left| \frac{u}{au+b} \right| + C$$

2.

$$\int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a} \ln \left| \frac{a+\sqrt{a^2-u^2}}{u} \right| + C$$

3.

$$\int \frac{du}{a^2-u^2} = \frac{1}{2a} \ln \left| \frac{a+u}{a-u} \right| + C$$

4.

$$\int \frac{du}{u\sqrt{a^2+u^2}} = -\frac{1}{a} \ln \left| \frac{a+\sqrt{a^2+u^2}}{u} \right| + C$$

5.

$$\int a^u du = \frac{a^u}{\ln a} + C$$