Research Summary

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I expect to receive my Ph.D. from the University of Notre Dame in May 2005 for my research in vertex operator algebra theory. My dissertation examines the coalgebraic structure arising from the geometry of genus-zero worldsheets in conformal field theory. I would like to continue this work and am particularly interested in extending the known structure of vertex operator algebras and their representations to bialgebraic and Hopf algebraic structures as a continuation of my work in extending them to the coalgebraic structure. I would also like to understand to what extent the conformal geometry underlying these structures is independent of the multiplicative and comultiplicative structures themselves.

Vertex algebras and vertex operator algebras (VOAs) are one of the exciting new fields in modern algebra and, although much progress has been made since their inception, their intricate structure has made vertex algebras and VOAs two of the more challenging mathematical objects to classify and generalize. The definitions of vertex algebra and of VOA were partly motivated by the structures one would expect from genus-zero conformal field theory. VOAs are also connected to understanding the Monster group (the largest sporadic finite simple group), representation theory on infinite-dimensional Lie algebras, modular functions and modular forms, Calabi-Yau manifolds, infinite-dimensional integrable systems, knot and three-manifold invariants and elliptic cohomology. Among other accolades the field has received, Borcherds received the Fields medal in 2000 for his work in monstrous moonshine, a collection of mysterious connections between the theory of VOAs and modular functions in number theory. A deeper and broader understanding of VOAs would not only be a benefit the larger mathematics community, but also much of the theoretical physics community.

As a student of Katrina Barron's at Notre Dame, I have developed the algebraic theory arising from the path swept out by splitting strings in space-time. My dissertation examines the geometric motivation arising from the model of one string splitting in space-time and the induced comultiplicative structure, motivating a structure I called a vertex operator coalgebra (VOC). I have also constructed a class of examples of VOCs, discussed an algebraic correspondence between certain VOCs and VOAs, defined a more general notion called a vertex coalgebra, and described the derivation, commutativity, associativity, skew-symmetry and modules properties of both vertex coalgebras and VOCs.

My past research opens doors for further investigation in a number of directions. First, from a purely algebraic perspective, having an algebraic structure (or in our case a multiplicative structure that satisfies a kind of generalization of the classical Jacobi identity) and a coalgebraic structure (similarly a comultiplicative structure satisfying a different Jacobi type identity) cries out to have the bialgebraic theory developed (particularly in the vertex tensor categories setting). Second, having a multiplication and a comultiplication allows for the consideration of antipodal maps (in the Hopf algebraic sense) and investigation of the corresponding module theory. This question can actually be considered independent of the first but answers to both obviously fit together well, and might give an interpretation to tensor products of VOA modules in the same way tensors of Lie algebra or group ring modules may be understood in terms of their corresponding Hopf algebras. Third, it would be useful to establish how much vertex tensor categories and the corresponding module tensors of VOAs depend on the conformal geometry and to what degree these are independent structures.

In the long term, I hope to have an established and fruitful research program, as well as a successful teaching career that gives students an opportunity to enter research mathematics even at the undergraduate level. I would like to develop collaborative relationships in the algebraic, topological and geometric communities closest to conformal field theory, and broaden my exposure to new progress in the mathematical research community at large.