Definitions: A hyperbola is the set of all points in the plane for which the difference of the distances from two fixed points (called the foci) is constant.

The line containing the foci is called the transverse axis, and the midpoint of the segment joining the foci is called the center of the hyperbola. The line containing the center and that is perpendicular to the transverse axis is called the conjugate axis. The vertices are the intersection points of the hyperbola and the transverse axis.

Equation of a hyperbola: We can derive the equation of a hyperbola where the differences of the distances from \((c, 0)\) and \((-c, 0)\) is \(\pm 2a\), assuming \(c > a > 0\); note that this will place the center at the origin, the transverse axis will be the \(x\)-axis, and the conjugate axis will be the \(y\)-axis:

\[
\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad (where \quad c^2 = a^2 + b^2).
\]

Result of our derivation: The hyperbola with center \((0, 0)\), vertices \((\pm a, 0)\) and foci \((\pm c, 0)\) has equation of the form \(\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1\) (where \(c^2 = a^2 + b^2\)).

Going Deeper:

1. For the form above, where are the vertices located?
2. Why does a hyperbola exhibit asymptotic behavior?

3. The general form for the equation of a hyperbola that we derived has transverse axis horizontal. What will the equation be for a hyperbola with transverse axis vertical, assuming that our assumptions for $a$, $b$ and $c$ do not change? Where are the vertices located in this form? What are the equations of the asymptotes?

4. All of the above forms assume the center of the hyperbola is at the origin; what would these forms look like with center $(h, k)$? How does this change the locations of the transverse/conjugate axes and the vertices and asymptotes?

**Homework Exercises:**

1. Find the equation of the hyperbola with center at $(0, 0)$, focus at $(5, 0)$ and vertex at $(-3, 0)$.

2. Find the equation of the hyperbola with center at $(0, 0)$, focus at $(0, 6)$ and vertex at $(0, -2)$.

3. Find the vertices and foci for the hyperbola with equation $9x^2 - 4y^2 = 36$. Identify the transverse and conjugate axes.

4. Discuss the conic section given by $\frac{(x-3)^2}{4} - \frac{(y+2)^2}{25} = 1$ and graph. (Here, “discuss” means to describe the anatomy of the hyperbola. Give foci, center, vertices, equations of asymptotes, transverse and conjugate axes.)

5. Discuss the conic section given by $-4x^2 + y^2 - 16x - 2y - 19 = 0$ and graph.