

Name: \_\_\_\_\_

### Math 138 – Final Exam

Directions: Read each problem carefully. Remember to show all work in a logical and legible manner. To receive credit, **all answers must follow logically from the work that you show on your exam.** You may not share calculators or use the calculator function of a cell phone.

1. Find the half-life of radioactive iodine if, after 20 days, 0.53 kilograms of an initial 3 kilograms remains. The radioactive decay formula is  $y = ae^{-bt}$ .

2. Find the remaining factor(s) of  $f$  and use your results to write a complete factorization of  $f$ .

$$f(x) = 2x^3 + 3x^2 - 17x + 12, \quad \text{Factors: } (2x-3), (x-1).$$

Remaining factor = \_\_\_\_\_

Complete factorization of  $f$ : \_\_\_\_\_

3. Match the following functions with their domains. Fill in the blank with the correct letter (a-k) from the lists on the left. Not all letters will be used and some may be used more than once.

\_\_\_\_\_  $f(x) = 3x^3 - 3x + 3$

a.  $[-3, 3]$

f.  $[3, \infty)$

\_\_\_\_\_  $f(x) = \sqrt{x-3}$

b.  $(0, \infty)$

g.  $(-3, 3)$

\_\_\_\_\_  $f(x) = 3^{-x}$

c.  $[0, \infty)$

h.  $(-3, \infty)$

\_\_\_\_\_  $f(x) = \log_3 x$

d.  $(-\infty, \infty)$

j.  $(-\infty, -3)$

\_\_\_\_\_  $f(x) = \frac{3x^2}{x-3}$

e.  $(3, \infty)$

k. none of the above

4. Consider the points  $(4, -6)$  and  $(-2, 3)$ .
- Find the slope of the line passing through these points.
  - Find the equation of the line passing through these points. (Put your answer in slope-intercept form.)
5. A high school had an enrollment of 1200 students in 1995. During the next 10 years, the enrollment increased by approximately 50 students per year.
- Write a linear equation giving the enrollment  $N$  in terms of the year  $t$ . (Let  $t=5$  correspond to the year 1995.)
  - If this constant rate of growth continues, predict the enrollment in the year 2010.

6. Find all zeros of the following, using the methods learned in class.

a.  $f(x) = \frac{2}{x+3} + 2(x+1)$       Zeros are \_\_\_\_\_

b.  $f(x) = 2x^2 + 6x - 1$       Zeros are \_\_\_\_\_

7. Consider the equation  $y = x^2 - 6x + 5$ . Find the vertex, identify it is a minimum or maximum, and find the exact (not rounded) real zeros.

the vertex (      ,      ) is a \_\_\_\_\_

zeros:  $x =$  \_\_\_\_\_ , \_\_\_\_\_

8. Find the number of units that produces a maximum revenue  $R$  (in dollars) for  $R = 800x - 0.055x^2$  where  $x$  is the number of units produced.

9. Let  $f(x) = \sqrt{x+9}$ ,  $g(x) = x^2 + 2$ , and  $h(x) = 2x - 1$ . Find the following. Put your answers in the blanks.

a.  $f(0) =$  \_\_\_\_\_

b.  $g(x+2) =$  \_\_\_\_\_

c.  $(h-g)(x) =$  \_\_\_\_\_

d.  $(g \circ f)(x) =$  \_\_\_\_\_

e.  $(h \circ g)(-2) =$  \_\_\_\_\_

10. Solve the following system of equations using either substitution or elimination. (Choose **one** of the methods and show your work!)

$$7x + 4y = 2$$

$$5x + 3y = -4$$

11. One investment paid 8% per year and another paid 10% per year. If \$4,000 was split between the two and earned exactly \$368.73 total interest in a year, how much was invested at each rate?

Amount invested at 8% = \_\_\_\_\_

Amount invested at 10% = \_\_\_\_\_

12. Write the expression as a single logarithm and simplify as much as possible.

$$\log_7(2x) - [\log_7(x-1) + 2\log_7(x)]$$

13. Expand the logarithmic expression as much as possible.

$$\log_3\left(\frac{5x^3}{\sqrt{x-1}}\right)$$

14. Solve for  $x$ . Round to four decimal places if necessary. Put your answers in the blanks.

a.  $11 = 3^{(x+7)}$   $x = \underline{\hspace{2cm}}$

b.  $\ln \sqrt{x+1} = 2$   $x = \underline{\hspace{2cm}}$

c.  $\frac{600}{1 + e^{-x}} = 575$   $x = \underline{\hspace{2cm}}$

d.  $\log_3 x + \log_3(x-8) = 2$   $x = \underline{\hspace{2cm}}$

15. If after  $w$  weeks of training, Maria rides a horse at  $S = 60(1 - e^{-0.08w})$  km per hour, how many weeks will it take her to ride at 20 km per hour?

16. On the day a child was born, a lump sum was deposited in a trust fund paying 6.5% interest compounded continuously. How much money should have been deposited so that the trust will be worth 1,000,000 on the child's 25<sup>th</sup> birthday?

Bonus: A group of 642 CIA agents is captured by a terrorist cell. They plan on killing every other agent (every second agent) until only one is left. Which is the best seat to sit in?